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Abstract

In this paper, we present a global state feedback tracking controller for underactuated surface marine vessels. This controller is based on saturated control inputs and, under an assumption on the reference trajectory, the closed-loop system is globally asymptotically stable (GAS). It has been designed using a 3 Degree of Freedom benchmark vessel model used in marine engineering. The main feature of our controller is the boundedness of the control inputs, which is an essential consideration in real life. In absence of velocity measurements, the controller works and remains stable with observers and can be used as an output feedback controller. Simulation results demonstrate the effectiveness of this method.

Index Terms

Global tracking, bounded feedback, Lyapunov function, underactuated surface marine vessels.

I. Introduction

Precise tracking control of surface marine vessels (ships and boats) is often required in critical operations such as support around off-shore oil rigs [1]. This problem is of particular interest as marine vessels are often underactuated, i.e. the number of independent actuators is less than the degrees of freedom (DOF) to be controlled. In this paper, we consider the problem of tracking control of a 3-DOF vessel model (surge, sway and yaw [2]), working under two independent actuators capable of generating surge force and yaw moment only. It has been shown in [3], [4], [5] that under Brockett's necessary condition [3], stabilization of this system is impossible with continuous or discontinuous time-invariant state feedback. This can be seen in [6] where the authors developed a continuous time-invariant controller that achieved global exponential position tracking but the vessel orientation could not be controlled. In addition, it is shown in [7] that the underactuated ship can not be transformed into a driftless chained system; which means that the control techniques used for the similar problem of nonholonomic mobile robot control cannot be applied directly to the underactuated ship control. Accordingly, control of underactuated vessels in this configuration has been studied rigorously by contemporary researchers, examples of which are [8], [9], [10], [11], [12].

In [7], the author showed that under discontinuous time-varying feedback, the underactuated vessel is strongly accessible and small-time locally controllable at any equilibrium. A discontinuous time-invariant controller was proposed which showed exponential convergence of the vessel towards a desired equilibrium point, under certain hypotheses imposed on the initial conditions. In [1], a continuous periodic time-varying feedback controller was presented that locally exponentially stabilizes the system on the desired equilibrium point by using a global coordinate transformation to render the vessel's model homogenous. In [8], a combined integrator backstepping and averaging approach was used for tracking control, together with the continuous time-varying feedback controller for position and orientation control. This combined approach, later on used in [13], provides practical global exponential stability as the vessel converges to a neighborhood of the desired location or trajectory, the size of which can be chosen arbitrarily small. Jiang [14] used Lyapunov's direct method for global tracking under the assumption that the reference yaw velocity requires persistent excitation condition; therefore implying that a straight line trajectory could not be tracked. This drawback was overcome in [15] and [16]. Do et al. [15] proposed a Lyapunov based method and backstepping technique for stabilization and tracking of underactuated vessel. In this work, conditions were imposed on the trajectory to transform the tracking problem into dynamic positioning, circular path tracking, straight line tracking and parking.

In this paper, we address the global tracking control of underactuated vehicles, using saturated state feedback control. Our work addresses the remaining case not treated in [15], i.e., the yaw angle of the tracked trajectory does not admit a limit at time goes to infinity. This research is therefore in the same direction as in [17], where the author achieved practical stability. Our algorithm provides asymptotic convergence to the tracked trajectory from any initial point. The advantage of using saturated controls is that the global asymptotic stability is ensured while the control inputs remain bounded (as real life actuators are all limited in output). The proposed controller has been proven to work with state measurements, as well as with observers in the case where all states may not be measured.

The paper is organized as follows; the vessel model is presented in Section 2 and the control problem is formulated in Section 3. In Section 4, the controller is developed and the proof of stability is given. In Section 5, the stability of the controller is shown in presence of observation errors. Simulations are given in Section 6 and concluding remarks are presented in Section 7.

II. Vessel Model

In this section, we will first discuss the physical model of the marine vessel and the related assumptions on physical phenomena associated with its motion. Then, a mathematical reformulation will be presented, following variable and time-scale changes, to obtain a suitable form for control design.

A. Physical Model

The general 6-DOF rigid body model for surface marine vessels presented in [2] can be reduced by considering surge, sway and yaw motions only, under the following assumptions [17],

- $(H1)$ Heave, roll and pitch motions induced by drift forces of wind, wave and ocean current can be neglected.
- $(H2)$ The inertia, added mass and hydrodynamic damping matrices are diagonal (valid for ships having port/starboard and fore/aft symmetry).

The aft propeller configuration provides only the surge force τ_u and the yaw moment τ_r. The kinematic and dynamic equations of the vessel can therefore be written as

$$\left\{\begin{array}{rcl}\dot{x} & = & u\cos(\psi) - v\sin(\psi),\\ \dot{y} & = & u\sin(\psi) + v\cos(\psi),\\ \dot{\psi} & = & r,\\ \dot{u} & = & \frac{1}{c}vr - au + \bar{\tau}_1,\\ \dot{v} & = & -cur - bv,\\ \dot{r} & = & \kappa uv - dr + \bar{\tau}_2,\end{array}\right. \tag{1}$$

where (x,y) and ψ are the coordinates and the yaw angle of the vessel in the earth-fixed frame, and u, v and r denote the surge, sway and yaw velocities respectively. The control inputs $\bar{\tau}_1$ and $\bar{\tau}_2$ are the normalized expressions of the surge force and yaw moment, given as

$$\bar{\tau}_1 = \frac{1}{m_1}\tau_u,\ \bar{\tau}_2 = \frac{1}{m_3}\tau_r. \tag{2}$$

The parameters a, b, c, d and κ are positive constants that represent the mechanical properties of the system, namely the inertia $m_i > 0$ and hydrodynamic damping d_i, where $i = 1,\ 2,\ 3$ corresponds to surge, sway and yaw motions respectively. The constants are defined as follows

$$a = \frac{d_1}{m_1},\ b = \frac{d_2}{m_2},\ c = \frac{m_1}{m_2},\ d = \frac{d_3}{m_3},\ \kappa = \frac{m_1 - m_2}{m_3}. \tag{3}$$

B. Model for control

For control design, the system model (1) can be simplified by normalizing the physical parameters through straightforward variable and time-scale changes. For the sake of clarity, let us rewrite System (1) as follows,

$$(\bar{S}) \left\{ \begin{array}{rcl} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} & = & R_\psi \begin{pmatrix} u \\ v \end{pmatrix}, \\ \dot{\psi} & = & r, \\ \begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} & = & -D_0 \begin{pmatrix} u \\ v \end{pmatrix} - rA_c \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{\tau}_1, \\ \dot{r} & = & \kappa uv - dr + \bar{\tau}_2, \end{array} \right. \tag{4}$$

where the matrices D_0, R_ψ and A_c are given as

$$D_0 = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix},\ R_\psi = \begin{pmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{pmatrix},\ A_c = \begin{pmatrix} 0 & -1/c \\ c & 0 \end{pmatrix}. \tag{5}$$

Let us consider the following matrices

$$A_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},\ D_\rho = \begin{pmatrix} \rho & 0 \\ 0 & \rho c \end{pmatrix}, \tag{6}$$

where ρ is a positive constant to be chosen later. Then we obtain

$$A_1 = D_\rho^{-1} A_c D_\rho. \tag{7}$$

Linear changes of variables and time-scale are introduced in System (4) as follows

$$\begin{array}{rcl} s & = & dt, \\ \begin{pmatrix} x(s) \\ y(s) \end{pmatrix} & = & \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \\ \psi(s) & = & \psi(t), \\ \begin{pmatrix} u(s) \\ v(s) \end{pmatrix} & = & \frac{1}{d} D_\rho^{-1} \begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \\ r(s) & = & \frac{r(t)}{d}. \end{array} \tag{8}$$

From this variable and time-scale change, it can be deduced by simple calculations that under the following definitions,

$$\beta = \frac{\kappa}{c\rho^2}, \quad D = \frac{D_0}{d} =: \begin{pmatrix} a_1 & 0 \\ 0 & b_1 \end{pmatrix}, \quad \tau_1 = \frac{\bar{\tau}_1}{\rho d^2}, \quad \tau_2 = \frac{\bar{\tau}_2}{d^2}, \tag{9}$$

the dynamics of the vessel, denoted by (S), can be rewritten as follows,

$$(S) \begin{cases} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} &= R_\psi D_\rho \begin{pmatrix} u \\ v \end{pmatrix}, \\ \begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} &= -D \begin{pmatrix} u \\ v \end{pmatrix} - rA_1 \begin{pmatrix} u \\ v \end{pmatrix} + \tau_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \\ \dot{\psi} &= r, \\ \dot{r} &= \beta uv - r + \tau_2, \end{cases} \tag{10}$$

III. Problem Formulation

The goal of this paper is tracking control of the presented underactuated marine vessel by controlling its position and orientation. The vessel is forced to follow a reference trajectory which is generated by a 'virtual vessel', as follows,

$$(S_{re}) \begin{cases} \begin{pmatrix} \dot{x}_{re} \\ \dot{y}_{re} \end{pmatrix} &= R_{\psi_{re}} D_\rho \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix}, \\ \begin{pmatrix} \dot{u}_{re} \\ \dot{v}_{re} \end{pmatrix} &= -D \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} - r_{re}A_1 \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tau_{1,re}, \\ \dot{\psi}_{re} &= r_{re}, \\ \dot{r}_{re} &= \beta u_{re} v_{re} - r_{re} + \tau_{2,re}, \end{cases} \tag{11}$$

where all variables have similar meanings as in System (10). Tracking control is achieved by using saturated control inputs and under the assumption that the velocities are bounded [18], [19]. This assumption holds true physically as resistive drag forces increase as the velocity increases and therefore the latter cannot increase indefinitely if the control is bounded. These assumptions are also valid for the reference system and are formalized in the following manner:

Assumption 1. *There exist constraints on the control inputs and velocities such that*

$$\begin{array}{l} |\bar{\tau}_1| \leq \bar{\tau}_{1,\max}, \ |\bar{\tau}_2| \leq \bar{\tau}_{2,\max}, \\ |u| \leq \bar{u}_{\max}, \ |v| \leq \bar{v}_{\max}, \end{array} \tag{12}$$

where $\bar{\tau}_{1,\max}$, $\bar{\tau}_{2,\max}$, $\bar{u}_{\max}$ and $\bar{v}_{\max}$ are known positive constants.

Assumption 2. *The velocities u_{re}, v_{re} and the forces $\tau_{1,re}$ and $\tau_{2,re}$ are bounded as follows,*

$$\begin{array}{l} |\tau_{1,re}| \leq \tau_{1,\max}, \ |\tau_{2,re}| \leq \tau_{2,\max}, \\ |u_{re}| \leq \bar{u}_{\max}, \ |v_{re}| \leq \bar{v}_{\max}, \end{array} \tag{13}$$

and the reference angle ψ_{re} does not converge to a finite limit as t tends towards infinity.

The variable and time-scale change defined in the previous section requires the following new bounds to be defined for the new control inputs τ_1 and τ_2, denoted by $\tau_{1,max}$ and $\tau_{2,max}$ respectively:

$$\tau_{1,max} = \frac{\bar{\tau}_{1,max}}{\rho d^2}, \quad \tau_{2,max} = \frac{\bar{\tau}_{2,max}}{d^2}. \tag{14}$$

We consider the following condition upon the saturation limits of the control inputs, to be used later on in the control design. We use here m_1 to denote $\min(a_1/2, b_1)$.

$$\textbf{C1: } \beta\frac{\tau_{1,max}^2}{a_1 m_1} < \tau_{2,max}. \tag{15}$$

Note that this condition is always satisfied by an appropriate choice of the parameter ρ. Our control objective is that (S) follows (S_{re}). With respect to the frame of reference of the reference trajectory (S_{re}), the error system is defined as

$$\begin{pmatrix} e_x \\ e_y \\ e_u \\ e_v \\ e_\psi \\ e_r \end{pmatrix} = \begin{pmatrix} cos(\psi_{re}) & sin(\psi_{re}) & 0 & 0 & 0 & 0 \\ -sin(\psi_{re}) & cos(\psi_{re}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x - x_{re} \\ y - y_{re} \\ u - u_{re} \\ v - v_{re} \\ \psi - \psi_{re} \\ r - r_{re} \end{pmatrix}. \tag{16}$$

Defining new controllers w_1 and $\tilde{w}_2$, as follows:

$$\begin{aligned} w_1 &= \tau_1 - \tau_{1,re}, \\ \tilde{w}_2 &= \tau_2 - \tau_{2,re}, \end{aligned} \tag{17}$$

the dynamics of system (16) become

$$(S_e)\begin{cases} \begin{pmatrix} \dot{e}_x \\ \dot{e}_y \end{pmatrix} = -r_{re}A_1\begin{pmatrix} e_x \\ e_y \end{pmatrix} + D_\rho \begin{pmatrix} e_u \\ e_v \end{pmatrix} + sin(e_\psi)A_1 D_\rho \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} \\ \qquad + \left(cos\left(e_\psi\right) - 1\right) D_\rho \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + \left(R_{e_\psi} - Id_2\right)\begin{pmatrix} e_u \\ e_v \end{pmatrix}, \\ \begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} = -D\begin{pmatrix} e_u \\ e_v \end{pmatrix} - r_{re}A_1\begin{pmatrix} e_u \\ e_v \end{pmatrix} - e_r A_1 \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} w_1 + e_r\begin{pmatrix} -e_v \\ e_u \end{pmatrix}, \\ \dot{e}_\psi = e_r, \\ \dot{e}_r = \beta\left(uv - u_{re}v_{re}\right) - e_r + \tilde{w}_2. \end{cases} \tag{18}$$

The control objective therefore, is to force the error system (S_e) to zero, using the control variables w_1 and $\tilde{w}_2$.

IV. Controller design

We first develop the following intermediate result, concerning the bounds of u, v, r.

Lemma 1. *The variables u, v, r are bounded and satisfy*

$$\begin{aligned} \limsup_{t\to\infty} \|(u,v)\| &\leq \frac{\tau_{1,\max}}{2\sqrt{a_1 m_1}}, \\ \limsup_{t\to\infty} |r| &\leq \tau_{2,\max} + \beta\frac{\tau_{1,\max}^2}{2a_1 m_1}. \end{aligned} \tag{19}$$

Proof: Let us consider

$$\begin{pmatrix} u \\ v \end{pmatrix}^T \begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} \begin{array}{ll} = & u\dot{u} + v\dot{v} \\ = & u\left(-a_1 u + rv + \tau_1\right) + v\left(-b_1 v - ru\right) \\ \leq & -\dfrac{a_1}{2}u^2 - b_1 v^2 + \dfrac{\tau_1^2}{2a_1} \leq -m_1 V + \dfrac{\tau_1^2}{2a_1}. \end{array} \tag{20}$$

We at once deduce the first inequality in (19). In the same way, let us consider the following equation,

$$r\dot{r} = -r^2 - r\left(\tau_2 + \beta uv\right) = -r(r + \tau_2 + \beta uv),$$

from which we derive the second inequality in (19). ■

Remark 1. *As the reference trajectory system is similar to the vessel model, it can be shown that the limits defined in Lemma 1 are valid for u_{re}, v_{re}, r_{re} as well.*

We define a new control variable

$$w_2 = \beta\left(uv - u_{re}v_{ref}\right) + \tilde{w}_2. \tag{21}$$

As the upper bounds of u, v, u_{re} and v_{ref} are known according to Lemma 1 and Remark 1, we obtain

$$\limsup_{t\to\infty} \beta\left|uv - u_{re}v_{ref}\right| \leq \beta\frac{\tau_{1,max}^2}{a_1 m_1}. \tag{22}$$

If the control variable w_2 is bounded by a positive constant U_2, then the following constraint on U_2 must be satisfied:

$$U_2 + \beta\frac{\tau_{1,max}^2}{a_1 m_1} \leq \tau_{2,max}. \tag{23}$$

The existence of U_2 is satisfied according to Condition C1, given in (15),

$$0 < U_2 \leq \tau_{2,max} - \beta\frac{\tau_{1,max}^2}{a_1 m_1}. \tag{24}$$

With these preliminaries established, we will now proceed to fulfill the control objective by using the bounded controls w_1 and w_2. Considering $\sigma(.)$ as the standard saturation function, the main result is presented as follows,

Theorem 1. *If Assumption 1, 2 and Condition **C1** are fulfilled, then for an appropriate choice of constants U_1, ρ_1, ξ, M, U_2, k_1, k_2, μ, the following controller ensures global asymptotic stability of the tracking error system (S_e):*

$$w_1 = -U_1\sigma\left(\frac{\xi e_u}{U_1}\right) - \rho_1\sigma\left(MW_1\right), w_2 = -U_2\sigma\left(\frac{k_1}{U_2}e_\psi + \frac{k_2 - 1}{U_2}e_r\right), \tag{25}$$

with $W_1 = e_x + \dfrac{1}{\mu}e_u$.

A. *Proof of Theorem 1*

As the demonstration is long, the proof of this theorem and the choice of related constants are developed progressively.

We first consider the errors e_ψ and e_r. The control input w_2 is chosen as

$$w_2 = -U_2 \sigma\left(\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right), \tag{26}$$

where k_1 and k_2 are sufficiently large positive constants. Then, the dynamics of e_ψ and e_r in (S_e) become,

$$\begin{aligned} \dot{e}_\psi &= e_r, \\ \dot{e}_r &= -e_r - U_2 \sigma\left(\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right). \end{aligned} \tag{27}$$

Lemma 2. *If $U_2 > 0$ and $k_1 > k_2 - 1 > 0$, then after a sufficiently large time, the saturated control operates in its linear region and the errors e_ψ and e_r converge to zero exponentially.*

Proof: Let us consider the following Lyapunov function V,

$$V = \frac{\alpha}{2} e_r^2 + S\left(\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right), \tag{28}$$

where $S(\xi)$ is a positive definite function, defined by

$$S(\xi) = \int_0^\xi \sigma(s) ds, \tag{29}$$

and $\alpha = \dfrac{k_1 - k_2 + 1}{U_2^2}$ is a positive constant. The derivative of the Lyapunov function is

$$\begin{aligned} \dot{V} &= \alpha e_r \dot{e}_r + \sigma\left(\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right)\left[\frac{k_1}{U_2} \dot{e}_\psi + \frac{k_2-1}{U_2} \dot{e}_r\right], \\ &= \alpha e_r \left(-e_r - U_2 \sigma(.)\right) + \sigma(.)\left[\frac{k_1}{U_2} e_r + \frac{k_2-1}{U_2}\left(-e_r - U_2 \sigma(.)\right)\right], \\ &= -\alpha e_r^2 - (k_2-1)\sigma^2(.) + e_r \sigma(.)\left[-U_2 \alpha + \frac{k_1}{U_2} - \frac{k_2-1}{U_2}\right], \\ &= -\alpha e_r^2 - (k_2-1)\sigma^2\left(\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right). \end{aligned} \tag{30}$$

The derivative $\dot{V}$ is negative for $(e_\psi, e_r) \neq (0,0)$; and after a finite time we obtain

$$\left|\frac{k_1}{U_2} e_\psi + \frac{k_2-1}{U_2} e_r\right| \leq 1.$$

Then , the dynamics of e_ψ and e_r become:

$$\begin{pmatrix} \dot{e}_\psi \\ \dot{e}_r \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k_1 & -k_2 \end{pmatrix} \begin{pmatrix} e_\psi \\ e_r \end{pmatrix} \tag{31}$$

As k_1 and k_2 are positive constants, System (31) is exponentially stable. ■

Lemma 2 shows that the errors e_ψ and e_r converge to zero under the control w_2. We will now consider the errors e_u and e_v. We choose the constants μ and ξ such that

$$\begin{pmatrix} a_1+\xi & 0 \\ 0 & b_1 \end{pmatrix} = \mu \begin{pmatrix} \rho & 0 \\ 0 & \rho c \end{pmatrix}. \tag{32}$$

This implies that

$$\xi = \frac{b_1}{c} - a_1, \ \mu > 0. \tag{33}$$

Lemma 3. *Consider the dynamics of e_u and e_v presented in Equation* (18)*. If the positive constants U_1 and ρ are chosen as*

$$\begin{aligned} a_1 &> U_1 + \rho, \\ U_1 &> \frac{\left|a_1 - \frac{b_1}{c}\right|}{\min\left(a_1, \frac{b_1}{c}\right)}\rho, \end{aligned} \tag{34}$$

then the control

$$w_1 = -U_1\sigma\left(\frac{\xi e_u}{U_1}\right) - \rho\sigma_1(.), \tag{35}$$

with $\sigma_1(.)$ to be chosen later, ensures that e_u and e_v are bounded, satisfying the following inequalities:

$$\limsup_{t\to\infty} \|(e_u, e_v)\| \leqslant \frac{\rho}{\sqrt{m_2\tilde{a}}}, \tag{36}$$

where

$$\tilde{a} = \inf_{t>0}\left(a_1 + \xi\frac{\sigma\left(\frac{\xi e_u}{U_1}\right)}{\frac{\xi e_u}{U_1}}\right) > 0, \quad m_2 := \min(\tilde{a}/2, b_1)., \tag{37}$$

Proof: Considering

$$\begin{pmatrix} e_u \\ e_v \end{pmatrix}^T \begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} = -\begin{pmatrix} e_u \\ e_v \end{pmatrix}^T D \begin{pmatrix} e_u \\ e_v \end{pmatrix} - e_r \begin{pmatrix} e_u \\ e_v \end{pmatrix}^T A_1 \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + e_u w_1. \tag{38}$$

By applying the control w_1 presented in (35), we get

$$\begin{pmatrix} e_u \\ e_v \end{pmatrix}^T \begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} \leq -a_1 e_u^2 - b_1 e_v^2 - U_1 e_u \sigma\left(\frac{\xi e_u}{U_1}\right) - \rho e_u \sigma_1(.) + C_0 |e_r| \sqrt{e_u^2 + e_v^2}, \tag{39}$$

where C_0 is a positive constant defined below,

$$C_0 = \bar{u}_{max} + \bar{v}_{max}.$$

According to Lemma 2, e_r tends to zero. This means that after a sufficiently large time,

$$\begin{pmatrix} e_u \\ e_v \end{pmatrix}^T \begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} \leq -\left[a_1 + \xi s(t)\right] e_u^2 - b_1 e_v^2 - \rho e_u \sigma_1(.). \tag{40}$$

Notice that $\tilde{a} > 0$ since it is trivially the case if $\xi \geq 0$, and otherwise, $\tilde{a} \geq a_1 + \xi = \dfrac{b_1}{c}$. Let us consider $\tilde{a}$ defined in equation (37). In this case,

$$\begin{pmatrix} e_u \\ e_v \end{pmatrix}^T \begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} \leq -m_2(e_u^2 + e_v^2) + \frac{\rho^2\sigma_1^2(\cdot)}{\tilde{a}}. \tag{41}$$

We immediately deduce (36). ∎

Lemma 3 proves the convergence of e_u and e_v to a neighborhood of zero. Since $\tilde{a} \geq \min\left(a_1, \frac{b_1}{c}\right)$, one gets together with condition (34) that

$$\limsup_{t\to\infty}\left|\frac{\xi e_u}{U_1}\right| \quad < \quad 1,$$

and after a finite time interval, the controller will exit saturation and enter its linear region of operation. We get

$$\sigma\left(\frac{\xi e_u}{U_1}\right) = \frac{\xi e_u}{U_1},$$

and the dynamics of e_u and e_v become

$$\begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} \quad = \quad -\mu D_\rho \begin{pmatrix} e_u \\ e_v \end{pmatrix} - r_{re} A_1 \begin{pmatrix} e_u \\ e_v \end{pmatrix} - \rho\sigma_1(.) \begin{pmatrix} 1 \\ 0 \end{pmatrix} - e_r A_1 \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + e_r \begin{pmatrix} -e_v \\ e_u \end{pmatrix}. \tag{42}$$

This result will be used further on during the study of convergence of e_x and e_y. Let us consider an intermediate variable $W = (W_1, W_2)^T$, such that

$$W = \begin{pmatrix} e_x \\ e_y \end{pmatrix} + \frac{1}{\mu}\begin{pmatrix} e_u \\ e_v \end{pmatrix}. \tag{43}$$

Based on the result in (42), we obtain the following result concerning W.

Lemma 4. *Consider the variable W defined in* (43) *and the controller w_1 presented in* (35) *with $\sigma_1(.) = \sigma(MW_1)$, where M is an arbitrary positive constant. Then*
W tends to a finite limit $\bar{W} = \begin{pmatrix} 0 \\ \bar{W}_2 \end{pmatrix}$.

Proof: The dynamics of W can be expressed as

$$\begin{aligned} \dot{W} \quad = \quad & -r_{re}A_1 W + r_{re}A_1 \frac{1}{\mu}\begin{pmatrix} e_u \\ e_v \end{pmatrix} + \sin\left(e_\psi\right) A_1 D_\rho \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + D_\rho \begin{pmatrix} e_u \\ e_v \end{pmatrix} \\ & + O\left(e_\psi^2, |e_\psi| \begin{pmatrix} e_u \\ e_v \end{pmatrix}\right) - D_\rho \begin{pmatrix} e_u \\ e_v \end{pmatrix} - r_{re}\frac{A_1}{\mu}\begin{pmatrix} e_u \\ e_v \end{pmatrix} - \frac{\rho}{\mu}\sigma_1(.)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ & + O\left(|e_r| \left(1 + \left\| \begin{matrix} e_u \\ e_v \end{matrix} \right\|\right)\right). \end{aligned} \tag{44}$$

In order to find the limsup of $\|W\|$, we calculate

$$\begin{aligned} W^T\dot{W} \quad & = \quad \sin\left(e_\psi\right) W^T A_1 D_\rho \begin{pmatrix} u_{re} \\ v_{re} \end{pmatrix} + W^T\left[O\left(e_\psi^2, |e_\psi|\begin{pmatrix} e_u \\ e_v \end{pmatrix} + O\left(|e_r|\right)\right)\right] - \frac{\rho}{\mu}W_1\sigma_1(.), \\ & = \quad O\left(\|W\| . \|e_\psi, e_r\|\right) - \frac{\rho}{\mu}W_1\sigma_1(.). \end{aligned} \tag{45}$$

From here, it is clear that

$$|W^T\dot{W}| \leq \|W\| \left[O\left(\|e_\psi, e_r\|\right) + \frac{\rho}{\mu}\right]. \tag{46}$$

Then there exists $A > 0$ such that the time derivative of $\|\dot{W}\|$ is bounded by a positive constant A and thus,

$$\|W\| \leqslant At + B, \tag{47}$$

where B is the initial value of $\|W\|$. As $\sigma_1(.) = \sigma(MW_1)$, this implies that

$$|W^T\dot{W}| + \frac{\rho}{\mu}W_1\sigma_1(MW_1) \leqslant O\left((At+B)e^{-ct}\right). \tag{48}$$

Then, the integral $\int_0^{\infty} |W^T(s)\dot{W}(s)|ds$ is finite , implying that$\|W\|$ is bounded, as well as the integral $\int\limits_0^{\infty} W_1\sigma(MW_1)ds$. As both W_1 and $\dot{W}_1$ are bounded, then according to Barbalat's Lemma, $W_1 \to 0$ as $t \to \infty$. Consequently W_2 tends towards a finite value $\bar{W}_2$ as $t \to \infty$. ■

The intermediate result obtained in the form of Lemma 4 permits us to further improve the result of Lemma 3, as follows:

Lemma 5. *If Lemmas 3 and 4 hold true, then the variables e_u and e_v converge to zero asymptotically.*

Proof: From Lemma 3 and setting $G(e_u, e_v) := (e_u^2 + e_v^2)/2$, Equation (41) can be rewritten as

$$\dot{G} + 2m_2G \leq \frac{\rho^2\sigma_1^2(MW_1)}{\tilde{a}}. \tag{49}$$

Since the right-hand side is integrable over $\mathbb{R}_+$, then one concludes using Barbalat's Lemma. ■

So far, we have established that the errors e_ψ, e_r, e_u and e_v converge to zero. From Lemmas 4 and 5, it can immediately be deduced that if $W_1 \to 0$ and $e_u \to 0$, then e_x will converge asymptotically to zero as well. We next address the convergence of the remaining error variable, e_y.

Lemma 6. *If Assumption 2 is satisfied, then $\bar{W}_2$ is equal to zero and e_y converges asymptotically to zero.*

Proof: From Equation (44) in Lemma 4, the dynamics of W can be expressed as follows,

$$\dot{W} = -r_{re}A_1W + O\left(\left|e_\psi\right|, |e_r|, W_1\sigma\left(MW_1\right)\right). \tag{50}$$

We define the new variable $\tilde{W}$ as follows,

$$\tilde{W} \triangleq \begin{pmatrix} \tilde{W}_1 \\ \tilde{W}_2 \end{pmatrix} = R_{\psi_{re}}W, \tag{51}$$

and the dynamics of $\tilde{W}$ are given by

$$\begin{aligned} \dot{\tilde{W}} &= \dot{R}_{\psi_{re}}W + R_{\psi_{re}}\dot{W}, \\ &= \dot{\psi}_{re}A_1R_{\psi_{re}}W - r_{re}R_{\psi_{re}}A_1W + R_{\psi_{re}}O\left(\left|e_\psi\right|, |e_r|, W_1\sigma\left(MW_1\right)\right), \\ &= O\left(\left|e_\psi\right|, |e_r|, W_1\sigma\left(MW_1\right)\right). \end{aligned} \tag{52}$$

With the exponential convergence of e_ψ and e_r to zero and the existence of $\int_0^t W_1\sigma(MW_1)ds$ when $t \longrightarrow \infty$, we get

$$\int_0^{+\infty} \left\|\dot{\tilde{W}}(s)\right\| ds < +\infty, \tag{53}$$

which means that $\tilde{W}$ will not explode and converge to a finite limit W_{re}, with

$$W_{re} = \begin{pmatrix} (W_{re})_1 \\ (W_{re})_2 \end{pmatrix}. \tag{54}$$

As demonstrated in Lemma 4, W_1 converges to zero and W_2 converges to $\bar{W}_2$. Then

$$\begin{array}{lcl} W_2 \sin(\psi_{re}) & \underset{t\to\infty}{\longrightarrow} & -(W_{re})_1\,, \\ W_2 \cos(\psi_{re}) & \underset{t\to\infty}{\longrightarrow} & (W_{re})_2\,. \end{array} \tag{55}$$

Using these two results, we proceed by contradiction. Assuming that $\bar{W}_2 \neq 0$, two cases are taken into account, $(W_{re})_2 = 0$ and $(W_{re})_2 \neq 0$.

- If $(W_{re})_2 = 0$, then $\cos(\psi_{re})$ converges to zero, i.e. ψ_{re} converges to a finite limit.
- If $(W_{re})_2 \neq 0$, then $\tan(\psi_{re}) = -\dfrac{(W_{re})_1}{(W_{re})_2} = constant$, i.e. ψ_{re} converges to a finite limit.

In either case, we find that ψ_{re} converges to a finite value, which contradicts Assumption 2. Therefore $\bar{W}_2$ has to be zero and according to Lemma 4, W tends to zero. Consequently, e_y converges asymptotically to zero. ∎

The proof of Theorem 1 demonstrates that the tracking errors between the vessel and the reference system, defined in terms of position coordinates, yaw angle and respective velocities, will converge asymptotically to zero. The controller will therefore, force the vessel (S) to follow the virtual vessel (S_{re}).

It should be noted that the controller presented in Theorem 1 has been designed under the assumption that all state variables are known. In the next section, the study is extended to the case where only the position and orientation states of the vessel are available and the velocities need to be observed.

V. Tracking without velocity measurement

In practical cases, only position and orientation feedbacks are available for navigation. Therefore the only available states of the vessel are x, y, ψ along with the the complete coordinate state set of the virtual vessel to be followed. For such output feedback systems, the variables u, v, r need to be observed. In this section, we will show that the controller presented in Theorem 1 is applicable in this case and the use of observation instead of measurement does not affect the stability.

We suppose that the velocities are obtained through an observer such as that presented by Fossen and Strand in [20], or a robust differentiator such as that presented in [21]. In both cases, an asymptotic convergence of observation errors is guaranteed. It can be noted that, when we use a differentiator, the estimated values $(\hat{u}, \hat{v}, \hat{r})$ of (u, v, r), can be determined according to the following equation

$$\begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} = D_\rho^{-1} R_{-\psi} \begin{pmatrix} \hat{\dot{x}} \\ \hat{\dot{y}} \end{pmatrix}, \quad \hat{r} = \hat{\dot{\psi}} \tag{56}$$

where $(\hat{\dot{x}}, \hat{\dot{y}}, \hat{\dot{\psi}})$ are the estimated values of $(\dot{x}, \dot{y}, \dot{\psi})$ respectively.

Let us follow the same steps used in the demonstration of stability of system (S_e) with velocity measurement. The observation error related to the velocity are define as below:

$$f_u = u - \hat{u}, \quad f_v = v - \hat{v}, \quad f_r = r - \hat{r}. \tag{57}$$

As the references are common, the observation errors can be described in terms of trajectory pursuit errors as

$$f_u = e_u - \hat{e}_u, \quad f_v = e_v - \hat{e}_v, \quad f_r = e_r - \hat{e}_r, \tag{58}$$

where,

$$\hat{e}_u = \hat{u} - u_{re}, \quad \hat{e}_v = \hat{v} - v_{re}, \quad \hat{e}_r = \hat{r} - r_{re}. \tag{59}$$

We note that the variable x, y, ψ are measured and the related observation errors are null.
The problem is transformed to demonstrate the stability of the error system (S_e) under control laws w_1 and $\tilde{w}_2$, which are now based on the observed values.
As in the previous case, we define

$$w_2 = \beta\left(\hat{u}\hat{v} - u_{re}v_{re}\right) + \tilde{w}_2. \tag{60}$$

Then the result of this section can be stated as the following theorem.

Theorem 2. *If Assumption 1, 2 and Condition **C1** are fulfilled, then for an appropriate choice of constants U_1, ρ, ξ, M, U_2, k_1, k_2, μ, the following controller ensures global asymptotic stability of the tracking error system (S_e):*

$$w_1 = -U_1\sigma\left(\frac{\xi\hat{e}_u}{U_1}\right) - \rho\sigma\left(M\hat{W}_1\right), w_2 = -U_2\sigma\left(\frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}\hat{e}_r\right), \tag{61}$$

with $\hat{W}_1 = e_x + \dfrac{1}{\mu}\hat{e}_u$.

Remark 2. *The choice of constants U_1, ρ, ξ, M, U_2, k_1, k_2, μ remain the same as in the case of Theorem 1, therefore their expressions and conditions will not be repeated in this section.*

Proof: The proof of Theorem 2, is largely based upon the proof of Theorem 1, and will be developed progressively in the same manner. We first consider the dynamics of error variables e_ψ and e_r:

$$\begin{aligned}
\dot{e}_\psi &= e_r, \\
\dot{e}_r &= \beta(uv - \hat{u}\hat{v}) - e_r - U_2\sigma\left(\frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}\hat{e}_r\right), \\
&= -e_r - U_2\sigma\left(\frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}e_r + f_1(t)\right) + f_2(t),
\end{aligned} \tag{62}$$

where,

$$\begin{aligned}
f_1(t) &= -\frac{k_2-1}{U_2}f_r, \\
f_2(t) &= \beta(uv - \hat{u}\hat{v}) \\
&= \beta(f_u\hat{v} + f_v\hat{u} + f_uf_v).
\end{aligned}$$

As mentioned before, the observer errors f_u, f_v and f_r converge asymptotically to zero and $(u,\ v)$ are bounded. This implies that $f_1(t)$ and $f_2(t)$ would converges to zero asymptotically.

Lemma 7. *If $f_1(t)$ and $f_2(t)$ converge to zero asymptotically, then for some large positive constants k_1 and k_2, System* (62) *is globally asymptotically stable.*

Proof: In order to prove Lemma 7, we consider the following Lyapunov function,

$$V = \frac{\alpha}{2}e_r^2 + S\left(\frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}e_r\right). \tag{63}$$

Let $z = \frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}e_r$, then the derivative of V, along System (62) can be calculated as follows,

$$\begin{aligned} \dot{V} &= \alpha e_r\dot{e}_r + \sigma\left(\frac{k_1}{U_2}e_\psi + \frac{k_2-1}{U_2}e_r\right)\left[\frac{k_1}{U_2}\dot{e}_\psi + \frac{k_2-1}{U_2}\dot{e}_r\right], \\ &= \alpha e_r\left(-e_r - U_2\sigma(z+f_1) + f_2\right) + \sigma(z)\left[\frac{k_1}{U_2}e_r + \frac{k_2-1}{U_2}\left(-e_r - U_2\sigma(z+f_1) + f_2\right)\right]. \end{aligned} \tag{64}$$

Using the well known inequality, $|ab| \leq \frac{a^2+b^2}{2}$, and taking $\alpha U_2 = \frac{k_1-k_2+1}{U_2} > 0$, we get

$$\dot{V} \leq -\frac{\alpha}{2}e_r^2 - (k_2-1)\,\sigma(z)\,\sigma(z+f_1) + \alpha U_2 e_r\left(\sigma(z) - \sigma(z+f_1)\right) + \frac{\alpha}{2}f_2^2 + \frac{k_2-1}{U_2}|f_2|. \tag{65}$$

From here, it is clear that

$$\begin{aligned} |\sigma(z) - \sigma(z+f_1)| &\leq |f_1|, \\ \sigma(z)\,\sigma(z+f_1) &\geq \sigma^2(z) - |f_1|, \end{aligned} \tag{66}$$

then,

$$\dot{V} \leq -\frac{\alpha}{2}e_r^2 - (k_2-1)\,\sigma^2(z) + (k_2-1)|f_1| + \alpha U_2|e_r||f_1| + \frac{\alpha}{2}f_2^2 + \frac{k_2-1}{U_2}|f_2|. \tag{67}$$

After a sufficiently large time interval T, it is assured that $|U_2 f_1| < \frac{1}{6}\ \forall t > T$, and

$$\dot{V} \leq -\frac{\alpha}{3}e_r^2 - (k_2-1)\sigma^2(z) + O\left(f_1^2, |f_2|, f_2^2\right) \tag{68}$$

From here, we obtain that

$$\limsup_{t\to\infty}|e_r| = \limsup_{t\to\infty}|\sigma(z)| = 0. \tag{69}$$

Therefore, System (62) is globally asymptotically stable. ∎

Lemma 7 shows that the errors e_ψ and e_r converge asymptotically to zero. Following the same steps as used in the previous section, we will now demonstrate the convergence of the error variables (e_u,e_v,e_x,e_y) of System (S_e). For e_u and e_v, Lemma 3 is modified using equations (32), (33) and (37), as follows:

Lemma 8. *Consider the dynamics of e_u and e_v presented in Equation* (18)*. Then, the control*

$$w_1 = -U_1\sigma\left(\frac{\xi\hat{e}_u}{U_1}\right) - \rho_1\sigma_1(.) \tag{70}$$

ensures that e_u and e_v are bounded and again verifiy the estimate (36).

Proof: The proof of this Lemma exactly follows the line of the argument of Lemma 3 with the controller w_1 written as:

$$w_1 = -U_1\sigma\left(\frac{\xi e_u}{U_1}+f\right) - \rho_1\sigma_1(.),$$

where $f = -\dfrac{\xi}{U_1} f_u$, converges to zero asymptotically, and by using the inequality $x\sigma(x+f) \geq x\sigma(x) - |x\sigma(f)|$.

∎

From this proof, it can be deduced that after a sufficiently large time interval, the dynamics of e_u and e_v become

$$\begin{pmatrix} \dot{e}_u \\ \dot{e}_v \end{pmatrix} = -\mu D_\rho \begin{pmatrix} e_u \\ e_v \end{pmatrix} - r_{re}A_1 \begin{pmatrix} e_u \\ e_v \end{pmatrix} - \rho_1\sigma_1(.)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + O\left(e_u\sigma(f_u), |e_r|\left(1+\left\| \begin{matrix} e_u \\ e_v \end{matrix} \right\|\right)\right). \tag{71}$$

Therefore the proof of convergence of W_1 to zero and W_2 to a finite limit, presented in Lemma 6, will hold true. ∎

From this discussion, it can be concluded that the replacement of known or measured states in the controller by observed states does not affect the stability of the closed-loop system.

VI. Simulation

The performance of the presented controller is illustrated by simulation. We apply the controller on an example of a monohull vessel, as considered in [15]. The length of this vessel is 32 m, and a mass of 118×10^3 kg. The parameters of the damping matrices as given as follow:

$$\begin{array}{lllllllll} d_1 & = & 215\times 10^2 Kgs^{-1}, & d_2 & = & 97\times 10^3 Kgs^{-1}, & d_3 & = & 802\times 10^4 Kgm^2s^{-1}, \\ m_1 & = & 120\times 10^3 Kg, & m_2 & = & 172.9\times 10^3 Kg, & m_3 & = & 636\times 10^5 Kgm^2. \end{array} \tag{72}$$

Based on these physical parameters, we find the parameters of System (1) as

$$a = 0.179,\ b = 0.561,\ c = 0.694,\ \beta = 0.126, \kappa = 8.32\times 10^{-4}. \tag{73}$$

Then, the parameters of the controller and the normalized system (S) are given by

$$\begin{array}{lllllllll} a_1 & = & 1.421, & b_1 & = & 4.449, & d & = & 0.126, \\ k_1 & = & 10, & k_2 & = & 10, & U_2 & = & 0.1, \\ U_1 & = & \frac{a_1}{2}, & \rho & = & \frac{a_1}{4}, & M & = & 0.1. \end{array} \tag{74}$$

The reference trajectory is generated by considering the surge force and the yaw moment as constants:

$$\tau_{1,re} = 10, \quad \tau_{2,re} = 0.05.$$

with the initial values

$$(x_{re}(0), y_{re}(0), \psi_{re}(0), u_{re}(0), v_{re}(0), r_{re}(0)) = \left(0\ m, 0\ m, 0\ rad,\ 0ms^{-1}, 0\ ms^{-1}, 0\ rads^{-1}\right)).$$

The initial conditions of the vessel are as follow:

$$(x(0), y_{re}(0), \psi(0), u(0), v(0), r(0)) = \left(50\ m, -150\ m, \frac{\pi}{4}\ rad,\ 50ms^{-1}, 0\ ms^{-1}, 0\ rads^{-1}\right).$$

The reference trajectory and the vessel are shown in a 2D coordinate plane in Figure 1. It can be seen that the vessel converges to the reference trajectory asymptotically. The convergence can also be seen in the position errors graph shown in Figure 2. The orientation error and its derivative also converge to zero, as seen in Figure 4. The convergence of e_u and e_v is shown in Figure 3. Figures 5 and 6 show the control signals τ_1 and τ_2 respectively. It is clear from these figures that the controllers are bounded. This is an essential property in real systems, where the control signals are constrained.

VII. Conclusion

In this paper, we have addressed the problem of tracking of an underactuated surface vessel with only surge force an yaw moment. The proposed controller ensures global asymptotic tracking of the vessel, following a reference trajectory modeled by a virtual vessel. It is also shown that the stability of the system is not affected if the state measurements are replaced by observers. The using of saturated inputs is essential as in real life the actuators have limited output. Simulation results illustrate the performance of the controller.

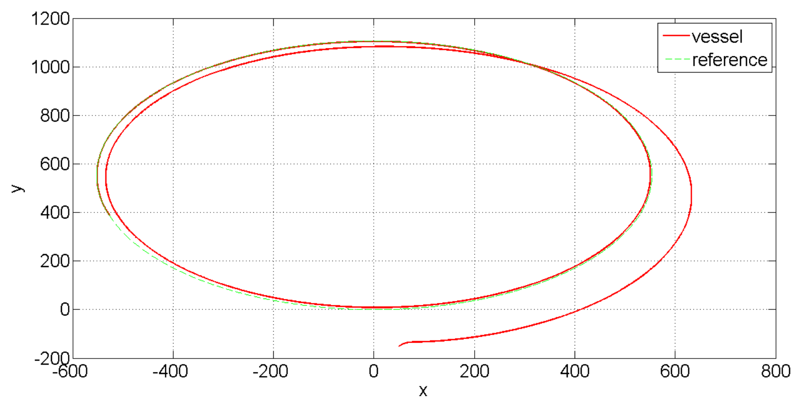

Fig. 1. Reference trajectory and the vessel

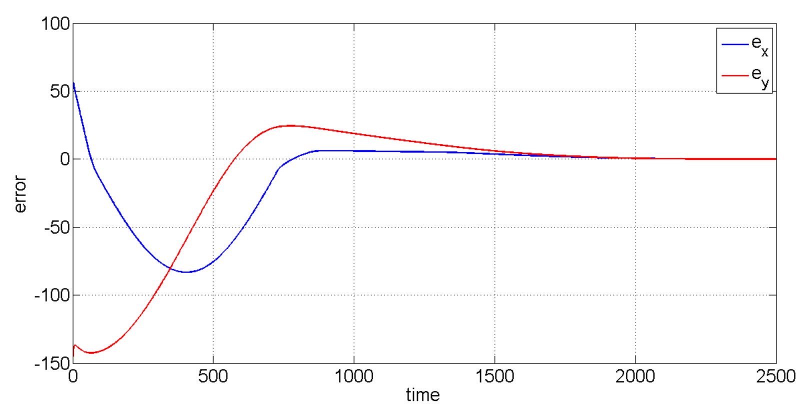

Fig. 2. Errors e_x and e_y

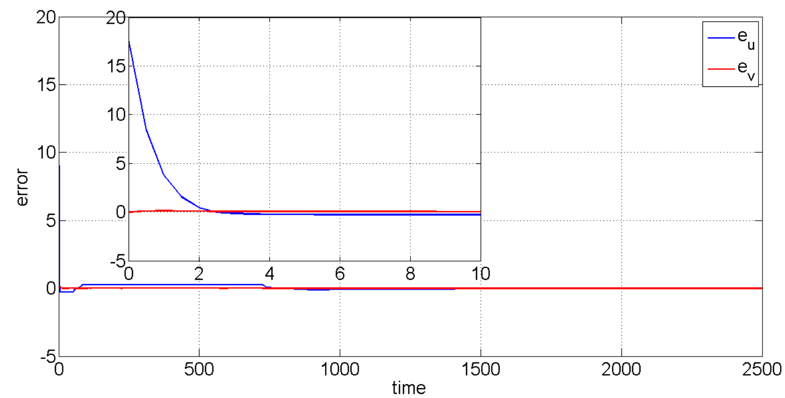

Fig. 3. Errors e_u and e_v

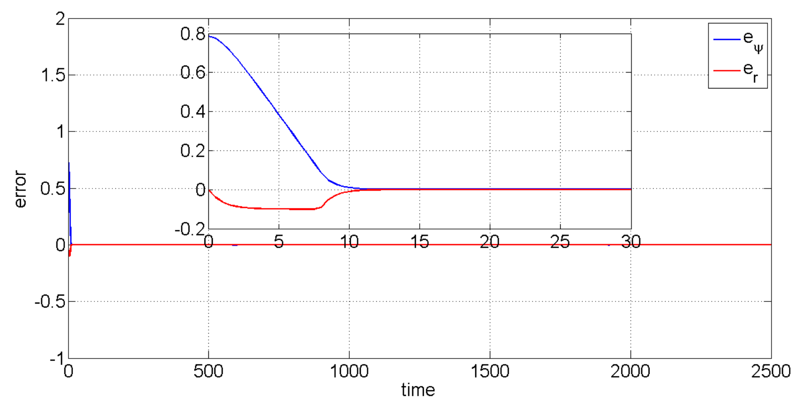

Fig. 4. Errors e_ψ and e_r

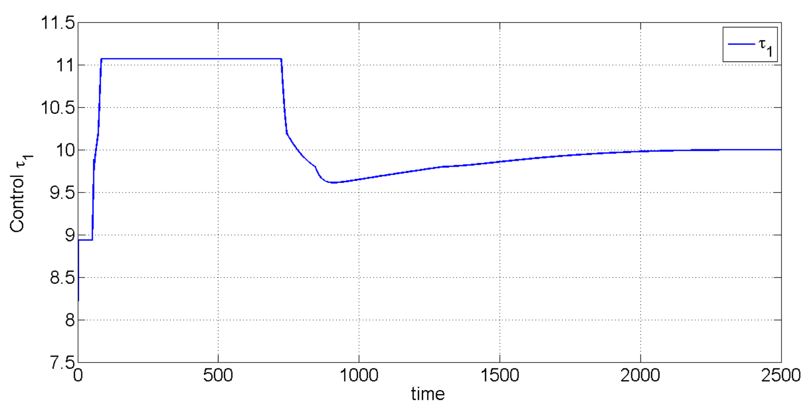

Fig. 5. Control τ_1

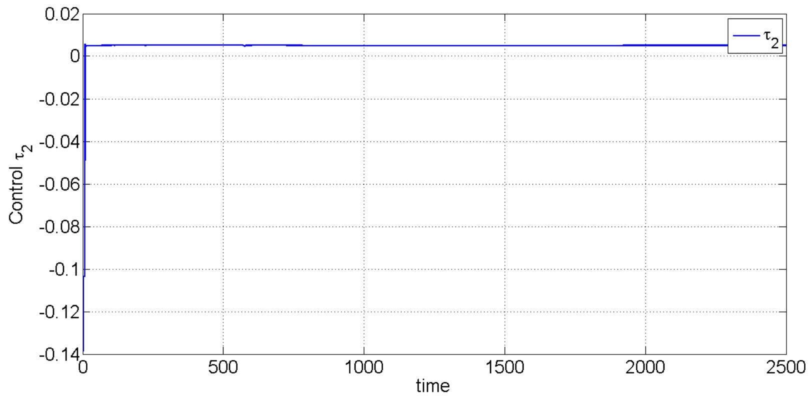

Fig. 6. Control τ_2

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